NAME ____________________

Student No ____________________

CONVEX OPTIMIZATION

DAYCLASS — Dr. Tom Luo

DURATION OF EXAMINATION: One week (due Next Friday)

Xidian University Midterm Examination — May 2013

This examination paper includes 3 pages and 3 problems. You are responsible for ensuring that your copy of the paper is complete. Bring any discrepancy to the attention of your invigilator.

Problem 1 (20 points)

Optimal facility location via second order cone programming

Suppose there are n locations in $\mathbb{R}^2$ whose coordinates are denoted by $a_i = (a_{i1}, a_{i2})^T$, $i = 1, 2, ..., n$. A facility is to be set up so that the weighted sum of Euclidean distances to all the given n locations is minimized. In other words, let $x \in \mathbb{R}^2$ denote the location of the facility. Then we wish to

$$\begin{array}{ll} \text{minimize} & \displaystyle\sum_{i=1}^{n} w_i \|x - a_i\|_2 \\ \text{subject to} & x \in \mathbb{R}^2, \end{array}$$

where $w_i > 0$ are given weights. Show that this problem can be converted to a second order cone program. If the L_2 norm $\|\cdot\|_2$ is replaced by L_1 norm or L_∞ norm, is the above problem convex? How would you reformulate the problem?

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Problem 2 (20 points)
Minimum length approximation.

Consider the problem

$$\begin{array}{ll} \text{minimize} & \text{length}(x) \\ \text{subject to} & \|Ax - b\| \leq \epsilon, \end{array}$$

with variable $x \in \mathbb{R}^n$, and problem parameters $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$, and $\epsilon > 0$. The objective function length(x) is defined as

$$\text{length}(x) = \min\{k \mid x_i = 0 \text{ for } i > k\}.$$

In a regression context, we are asked the find the minimum number of columns of A, taken in order, that can approximate the vector b within a tolerance ϵ.

Show that this is a quasiconvex optimization problem.

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Problem 3 (20 points)

Maximum volume box inside a polyhedron.

A *box* in $\mathbb{R}^n$ is a set of the form $B(l,u) = \{x \mid l \preceq x \preceq u\}$, where $u \succ l \in \mathbb{R}^n$. In this problem we consider the problem of finding the maximum volume box contained in a polyhedron $\mathcal{P} = \{x \mid Ax \preceq b\}$, where $A \in \mathbb{R}^{m\times n}$. A straightforward, but very inefficient, way to express the constraint $B(l,u) \subseteq \mathcal{P}$ is to use the set of $m2^n$ inequalities $Av^i \preceq b$, where v^i are the (2^n) corners of $B(l,u)$. (Clearly if the corners of a box lie inside a polyhedron, then the entire box does.) Fortunately it is possible to express the constraint in a far more efficient way.

Show that the constraint $B(l,u) \subseteq \mathcal{P}$ can be expressed as a set of m inequalities $A^+u - A^-l \preceq b$, where $A^+_{ij} = \max\{0, A_{ij}\}$ and $A^-_{ij} = \max\{0, -A_{ij}\}$. Note the extraordinary savings compared to expressing $B(l,u) \subseteq \mathcal{P}$ via the vertices, as described above — a factor of 2^n.

Use this result to pose the problem of maximizing the volume of the box as a convex optimization problem (that does not involve exponentially many constraints).